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A gauge covariant approximation to quantum electrodynamics

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Abstract. A non-perturbative method of solving the Dyson–Schwinger equations in QED, which *preserves* the gauge identities, is considered. The starting point is determined by an integral equation for the electron propagator spectral function which is explicitly solved in the Landau gauge; this determines the Green functions in successive orders of iteration since no spurious infinities arise beyond the usual renormalizable ones.

1. Introduction

A favourite pastime of theoreticians has been to look for approximate solutions of the complete set of equations linking the Green functions in various quantum field models like QED. Most of these approximations amount to summing specific sets of perturbation graphs with the foreknowledge or hope that the selection will provide the dominant contributions in the kinematic region of interest. In gauge theories most such approximations unfortunately violate the gauge constraints among the Green functions and this makes it difficult to judge the correctness or otherwise of the solutions found. There is one approximation method however which has the virtue of preserving the Ward identities at every stage: this is Salam's gauge technique (1963); by contrast, here it becomes difficult to judge the 'order of approximation' because the iteration procedure is basically non-perturbative. In early papers the gauge technique was applied to electrodynamics of mesons and spinors. The zeroth-order Green functions were obtained by truncating the Dyson–Schwinger equations so as to satisfy two-particle unitarity and could be simply calculated by applying first-order perturbation theory to the Lehmann spectral functions (Delbourgo and Salam 1964, Strathdee 1964). It then was verified that the resulting asymptotic behaviours of the Green functions were no different in the next order of iteration, signifying that the procedure was 'asymptotically stable'. However it remained unclear what the iterated sum of the series would yield and what bearing, if any, this has on renormalization group aspects of the problem (Manoukian 1974).

In this paper we wish to return to the gauge technique for QED but without resorting to two-particle unitarity for providing the starting point. Rather we shall solve the Dyson–Schwinger equation for the spinor propagator as a proper integral equation, neglecting photon dressing in the first place since that has no important bearing on the gauge identities. We shall then use the solution to determine its influence on the photon self-energy and other Green functions such as the vertex part. We do not look for finite electrodynamics and thus get no eigenvalue equation for e^2.

2. The zeroth gauge approximation

The Ward identities between Green functions in gauge theories are now well known (Nishijima 1960, Rivers 1966) even for non-Abelian gauges (Lee 1974, Kluberg-Stern and Zuber 1975). Thus with photon legs amputated the first few identities read

$$k^\mu S(p)\Gamma_\mu(p,p-k)S(p-k)=S(p-k)-S(p) \tag{1}$$

$$k^\mu S(p')\Gamma_{\nu\mu}(p'k';pk)S(p) = S(p')\Gamma_\nu(p',p'+k')S(p'+k')-S(p-k')\Gamma_\nu(p-k',p)S(p) \tag{2}$$

etc, where S denotes the complete electron propagator and Γ stands for the fully amputated connected Green functions with appropriate arguments and with coupling constants factorized out. In QED the propagators S of the electron and D of the photon, and the vertex part Γ_μ, play a central role via the Dyson–Schwinger equations

$$1=Z_\psi(\gamma\,.\,p-m+\delta m)S(p)-\mathrm{i}e^2Z_\psi\int \bar{\mathrm{d}}^4k\, S(p)\Gamma_\mu(p,p-k)S(p-k)\gamma_\nu D^{\mu\nu}(k) \tag{3}$$

$$D^{-1}_{\mu\nu}(k)=Z_A[-k^2\eta_{\mu\nu}+k_\mu k_\nu(1-a^{-1})]+\mathrm{i}e^2Z_\psi\,\mathrm{Tr}\int \bar{\mathrm{d}}^4p\gamma_\nu S(p)\Gamma_\mu(p,p-k)S(p-k) \tag{4}$$

$$\Gamma_\mu(p,p-k)=Z_\psi\gamma_\mu-\mathrm{i}e^2Z_\psi\int \bar{\mathrm{d}}^4p'K(p,p';p-k,p'-k)S(p')\Gamma_\mu(p',p'-k)S(p'-k) \tag{5}$$

in which we have adopted a covariant photon gauge parametrized by the bare constant a. One can make the gauge identities look more obvious by replacing (5) with

$$\Gamma_\mu(p,p-k)=Z_\psi\gamma_\mu-\mathrm{i}e^2Z_\psi\int \bar{\mathrm{d}}^4p'\gamma_\lambda S(p')\Gamma_{\nu\mu}(p'k';pk)D^{\lambda\nu}(k). \tag{5'}$$

Multiplication of (5') by k^μ yields (3) immediately, and in that sense incorporates it.

In the gauge technique (Salam 1963) one seeks solutions to equations (3)–(5) consistent with the Ward–Takahashi identities (1), (2), etc. To see how these can be determined iteratively, let us begin with the Lehmann spectral representation for the spinor propagator in the form†

$$S(p)=\left(\int_{-\infty}^{-m}+\int_m^\infty\right)\frac{\rho(W)\,\mathrm{d}W}{\gamma\,.\,p-W+\mathrm{i}\epsilon(W)0} \tag{6}$$

where $\rho(W)$ is a positive definite distribution in a non-gauge theory.

† The form

$$S(p)=\int_m^\infty\frac{(\gamma p\rho_1(s)+m\rho_2(s))\,\mathrm{d}s}{p^2-s+\mathrm{i}0}$$

with $m_0=Z\int m\rho_2(s)\,\mathrm{d}s$, $1=Z\int\rho_1(s)\,\mathrm{d}s$ is probably more familiar. The connection with the form (6) is provided by

$$\rho(W)=\epsilon(W)(W\rho_1(W^2)+m\rho_2(W^2)).$$

Since

$$S(p-k)-S(p)=\int \mathrm{d}W\rho(W)\frac{1}{\gamma\cdot p-W}\gamma\cdot k\frac{1}{\gamma\cdot(p-k)-W}$$

the simplest possible (but by no means the unique) solution of (1) is to take

$$S(p)\Gamma^{(0)}_{\mu}(p,p-k)S(p-k)=\int \mathrm{d}W\rho(W)\frac{1}{\gamma\cdot p-W}\gamma_{\mu}\frac{1}{\gamma\cdot(p-k)-W}. \tag{7}$$

One can of course add to (7) any arbitrary transverse function of the type $(k^2\eta_{\mu\nu}-k_{\mu}k_{\nu})F^{\nu}(p,p-k)$ which would have no effect on the gauge identities but we neglect this to begin with—a more precise reason follows shortly. Likewise a possible solution of identity (2) is provided by

$$S(p')\Gamma^{(0)}_{\nu\mu}(p'k';pk)S(p)$$
$$=\int \mathrm{d}W\rho(W)\frac{1}{\gamma\cdot p'-W}$$
$$\times\left(\gamma_{\nu}\frac{1}{\gamma\cdot(p'+k')-W}\gamma_{\mu}+\gamma_{\mu}\frac{1}{\gamma\cdot(p'-k)-W}\gamma_{\nu}\right)\frac{1}{\gamma\cdot p-W} \tag{8}$$

and so on. In analogy to (7) this is a weighted sum over electron mass distributions of the tree graphs. Indeed if we go to the mass shell of the charged spinor lines by picking out pole terms through the substitution $\rho(W)\to\delta(W-m)$ we get precisely the Born terms. We shall take this criterion as *defining* the zeroth gauge approximation $\Gamma^{(0)}$ of all the charged line Green functions; the photon lines are left undressed in this initial stage. Thus $\Gamma^{(0)}$ are functionals of the electron propagator S which is all we have to find, and this we can do by solving the electron line equation (3) as a true integral equation without resorting to two-particle unitarity. Using the basic $\Gamma^{(0)}$ we can then determine the photon propagator, vertex part and other connected functions in a recursive way† via

$$D^{-1(n+1)}=Z_Ak^{-2}+Z_{\psi}e^2\int S^{(n)}\Gamma^{(n)}S^{(n)}\gamma$$
$$S^{-1(n+1)}=Z_{\psi}(\gamma\cdot p-m_0)+Z_{\psi}e^2\int \Gamma^{(n)}S^{(n)}D^{(n)}\gamma \tag{9}$$
$$\Gamma^{(n+1)}=Z_{\psi}\gamma+Z_{\psi}e^2\int K[\Gamma^{(n)}]S^{(n)}\Gamma^{(n)}S^{(n)}$$

the hope being that the iterations will converge as $n\to\infty$ to the final true answers. Certainly for the iteration scheme to make sense it is necessary that higher-order multi-electron functions come out to be finite; otherwise all the advantages of conventional renormalizability would be lost and the gauge technique would become worthless. Finiteness is guaranteed when the stability criterion $\Gamma SD^{1/2}\sim 1/k^2$ is satisfied. We shall check *a posteriori* that our zeroth-order expressions do obey this asymptotic stability property and thus do not jeopardize renormalizability. Beyond this we have to look for some property of $\rho(W)$ which stands out as the iterations proceed since it is quite clear that the entire scheme is non-perturbative and we have not exactly

† Note that $S^{(1)}=S^{(0)}$ and $k\cdot\Gamma^{(1)}=k\cdot\Gamma^{(0)}$ in the first iteration.

expansions in e^2 to guide us about what we mean about the 'order of iteration'. Unfortunately we have not gone far enough in the iteration scheme to find out what it is except for the fact that the lowest-order spectral function (22) receives logarithmic corrections in succeeding orders.

3. The zeroth Green functions

In lowest order the equations (9) devolve to finding a solution of

$$\begin{aligned} Z_\psi^{-1} &= (\gamma \cdot p - m_0) S^{(0)}(p) - \mathrm{i}e^2 \int \bar{\mathrm{d}}^4 k\, S^{(0)}(p) \Gamma_\mu^{(0)}(p, p-k) S^{(0)}(p-k) D^{\mu\nu(0)}(k) \\ &= (\gamma \cdot p - m_0) \int \frac{\rho(W)\,\mathrm{d}W}{\gamma \cdot p - W} - \mathrm{i}e^2 \int \bar{\mathrm{d}}^4 k\, \mathrm{d}W \rho(W) \frac{1}{\gamma \cdot p - W} \gamma_\mu \\ &\quad \times \frac{1}{\gamma \cdot (p-k) - W} \gamma_\nu \cdot \left(-\eta^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}(1-a) \right) \frac{1}{k^2} \\ &= \int \frac{\rho(W)\,\mathrm{d}W}{\gamma \cdot p - W} (\gamma \cdot p - m_0 + \Sigma(p, W)) \end{aligned} \tag{10}$$

where $\Sigma(p, W)$ is obtained from lowest-order perturbation theory for an electron mass W. Thus

$$\operatorname{Im} \Sigma(p, W) = \frac{e^2(p^2 - W^2)}{16\pi p^3} [a(p^2 + W^2) - (a+3)pW] \theta(p^2 - W^2) \tag{11}$$

yielding $\Sigma(p, W)$ via a dispersion integral. Recalling that

$$1 = Z_\psi \int \rho(W)\,\mathrm{d}W \qquad \text{and} \qquad m_0 = Z_\psi \int W\rho(W)\,\mathrm{d}W, \tag{12}$$

let us perform our renormalizations on (10) and remove the pole term by putting $\rho(W) = \delta(W-m) + \sigma(W)$. Then

$$\frac{\Sigma(W, m)}{W-m} + \int (W' - m + \Sigma(W, W')) \frac{\sigma(W')\,\mathrm{d}W'}{W - W' + \mathrm{i}\epsilon(W')0} = 0 \tag{13}$$

with a once-subtracted

$$\Sigma(W, m) = \frac{(W-m)}{\pi} \int \frac{\operatorname{Im} \Sigma(W', m)\,\mathrm{d}W'}{(W'-m)(W-W')}. \tag{14}$$

Upon taking imaginary parts of (13) we remain with the integral equation

$$\epsilon(W)(W-m)\sigma(W) = \frac{\operatorname{Im} \Sigma(W, m)}{\pi(W-m)} + \int \mathrm{d}W'\, \sigma(W') \frac{\operatorname{Im} \Sigma(W, W')}{\pi(W-W')} \tag{15}$$

for the spectral function $\sigma(W)$. To show that exact solutions can be found let us specialize to the Landau gauge $a = 0$ where $\operatorname{Im} \Sigma$ is particularly easy† and the equation

† It is also relatively simple in the Yennie gauge $a = 3$ where to lowest order, $\sigma(W) = 3e^2(W^2 - m^2)\epsilon(W)/16\pi^2 W^3$ guarantees infrared finiteness but gives an ultraviolet divergence for Z_ψ.

reduces to $(W^2 \geqslant m^2)$

$$\epsilon(W)(W-m)W^2\sigma(W)$$
$$= -\frac{3e^2}{16\pi^2}\left(m(W+m)+\int^W \mathrm{d}W'\,\sigma(W')(W+W')W'\right).$$

Using dimensionless variables

$$\omega = W/m, \qquad \xi = -3e^2/16\pi^2, \qquad s(\omega)=\epsilon(\omega)\omega^2\sigma(\omega), \tag{16}$$

the equation simplifies to

$$\frac{(\omega-1)s(\omega)}{\xi} = (\omega+1)+\left(\int_1^{\omega\epsilon(\omega)} - \int_{-\omega\epsilon(\omega)}^{-1}\right)\mathrm{d}\omega'\,s(\omega')\left(1+\frac{\omega}{\omega'}\right). \tag{17}$$

This can in turn be reduced to a pair of coupled equations by making the substitution $s(\omega)=\omega s_1(\omega^2)+s_2(\omega^2)$. Thus

$$\begin{aligned}(s_2(\omega^2)-s_1(\omega^2))/\xi &= \int_1^{\omega^2}\mathrm{d}\omega'^2\,s_2(\omega'^2)/\omega'^2+1\\ (\omega^2 s_1(\omega^2)-s_2(\omega^2))/\xi &= \int_1^{\omega^2}\mathrm{d}\omega'^2\,s_1(\omega'^2)+1\end{aligned} \tag{18}$$

Finally the pair can be reconverted into hypergeometric equations

$$\begin{aligned}&\left(Z(1-Z)\frac{\mathrm{d}^2}{\mathrm{d}Z^2}-[1-Z(3-2\xi)]\frac{\mathrm{d}}{\mathrm{d}Z}-(1-\xi)^2\right)s_1(Z)=0\\ &\left(Z(1-Z)\frac{\mathrm{d}^2}{\mathrm{d}Z^2}-2(1-\xi)Z\frac{\mathrm{d}}{\mathrm{d}Z}+\xi(1-\xi)\right)s_2(Z)=0.\end{aligned} \tag{19}$$

The appropriate solutions, satisfying the boundary conditions embodied in (18) and incorporating an infrared† cut-off μ^2 are

$$s_1(Z)=\frac{2\xi}{(Z-1)}\left(\frac{Z-1}{\mu^2/m^2}\right)^{2\xi}F(\xi,\xi;2\xi;1-Z)$$
$$s_2(Z)=\frac{2\xi Z}{(Z-1)}\left(\frac{Z-1}{\mu^2/m^2}\right)^{2\xi}F(\xi,\xi+1;2\xi;1-Z).$$

In terms of the original variables this gives

$$\sigma(W)=\epsilon(W)\theta(W^2-m^2)\frac{2\xi}{W}\left(\frac{W^2-m^2}{\mu^2}\right)^{2\xi}\frac{m^2}{W^2-m^2}\left(F\left(\xi,\xi;2\xi;1-\frac{W^2}{m^2}\right)\right.$$
$$\left.+\frac{W}{m}F\left(\xi,\xi+1;2\xi;1-\frac{W^2}{m^2}\right)\right). \tag{20}$$

† We can verify the necessity of a μ^2 at the lower limit of integration if we attempt a perturbation expansion in ξ of equations (18). It is less obviously needed in the quoted solution (20) where we might even dispense with it by dropping μ^2/m^2 altogether.

The complete zeroth-order electron propagator follows:

$$S(p)=\frac{1}{\gamma.p-m}+\int\frac{\mathrm{d}W^2}{W^2}\frac{\gamma.p\,s_1(W^2/m^2)+ms_2(W^2/m^2)}{p^2-W^2+\mathrm{i}\epsilon}$$

$$=\frac{1}{\gamma.p-m}-\left(\frac{m^2}{\mu^2}\right)^{2\xi}\Gamma(1-\xi)\Gamma(1-\xi)\Gamma(1+2\xi)$$

$$\times\left[\frac{\gamma.p}{p^2}\left(F\left(1-\xi,1-\xi;1;\frac{p^2}{m^2}\right)-1\right)+\frac{1-\xi}{m}F\left(1-\xi,2-\xi;2;\frac{p^2}{m^2}\right)\right]\quad(21)$$

and the integral† for it converges comfortably since

$$\sigma(W)\sim\frac{\Gamma(1+2\xi)}{\Gamma(\xi)\Gamma(\xi)}\frac{m^{2-2\xi}W^{2\xi-3}}{\mu^{4\xi}}\left[1+\frac{m}{\xi W}+\mathrm{O}\left(\frac{1}{W^2}\right)\right].\quad(22)$$

Furthermore we may actually evaluate the electron renormalization constants in this gauge by going to asymptotic values of p in (21) or else from the formal expressions for the bare quantities:

$$Z_\psi^{-1}=1+\int\sigma(W)\,\mathrm{d}W=1+(m^2/\mu^2)^{2\xi}\Gamma(1-\xi)\Gamma(1-\xi)\Gamma(1+2\xi)\simeq2\qquad\text{for small }\xi$$

$$Z_\psi^{-1}m_0=m+\int W\sigma(W)\,\mathrm{d}W=m\left[1+\left(\frac{m^2}{\mu^2}\right)^{2\xi}\frac{\Gamma(1+2\xi)}{\Gamma(\xi)\Gamma(1+\xi)}\lim_{d\to0}\frac{\Gamma(d-\xi)\Gamma(d-\xi-1)}{\Gamma(d)}\right]$$

$$=m.$$

This ultraviolet finiteness is, of course, characteristic of the Landau gauge and not expected for other values of a.

The other Green function $\Gamma^{(0)}$ is given in this zeroth order by bare photon lines and by tree graphs weighted by the just found electron mass distribution. Expressions (7) and (8) are particular examples. For convergence the important point is the asymptotic behaviour (22).

4. First-order Green functions

In the next stage of the iteration we have to evaluate $D^{(1)}$ and the transverse part of $\Gamma^{(1)}$ using the lowest-order $D^{(0)}$, $\Gamma^{(0)}$ and $S^{(0)}$ just found. There are no new infinities because the asymptotic stability condition is amply satisfied so the Z will just perform their usual

† We have used the basic integral

$$\int_0^\infty x^{c-1}(x+y)^{-d}F(a,b;c,-x)\,\mathrm{d}x$$

$$=\frac{\Gamma(a-c+d)\Gamma(b-c+d)\Gamma(c)}{\Gamma(a+b-c+d)\Gamma(d)}F(a-c+d,b-c+d;a+b-c+d;1-y)$$

in this and succeeding expressions. Note the reality of σ in (20) and the fact that $S(p)$ correctly shows a cut for $p^2\geqslant m^2$ in formula (21) whose discontinuity is of course σ.

function. Concentrate first on the photon self-energy to first order,

$$\Pi^{(1)}_{\mu\nu}(k) = \mathrm{i}e^2 Z_\psi \,\mathrm{Tr} \int \bar{\mathrm{d}}^4 p \,\mathrm{d}W \rho(W) \left(\gamma_\nu \frac{1}{\gamma \cdot p - W} \gamma_\mu \frac{1}{\gamma \cdot (p-k) - W}\right)$$

$$= \mathrm{i}e^2 Z_\psi \int \mathrm{d}W \rho(W)(-k^2 \eta_{\mu\nu} + k_\mu k_\nu)\Pi(k^2, W^2) \tag{23}$$

where it is known from QED that

$$\Pi(k^2, m^2) = \frac{e^2}{12\pi^2}\left[\ln\frac{\Lambda^2}{m^2} - \frac{11}{6} - \frac{4m^2}{k^2} + \left(1 + \frac{2m^2}{k^2}\right)\left(1 - \frac{4m^2}{k^2}\right)^{1/2} \times \ln\left(\frac{[1-(4m^2/k^2)]^{1/2}+1}{[1-(4m^2/k^2)]^{1/2}-1}\right)\right]. \tag{24}$$

The renormalized propagator is thereby obtained as

$$D^{(1)-1}_{\mu\nu}(k) = (-k^2\eta_{\mu\nu} + k_\mu k_\nu)\left(1 + Z_\psi \int \mathrm{d}W \rho(W)(\Pi(k^2, W^2) - \Pi(0, W^2))\right) - k_\mu k_\nu Z_A/a. \tag{25}$$

Since

$$\Pi(k^2, W^2) - \Pi(0, W^2) = \frac{e^2}{12\pi^2}\left[-\frac{5}{3} - \frac{4W^2}{k^2} + \left(1 + \frac{2W^2}{k^2}\right)\left(1 - \frac{4W^2}{k^2}\right)^{1/2} \ln\left(\frac{[1-(4W^2/k^2)]^{1/2}+1}{[1-(4W^2/k^2)]^{1/2}-1}\right)\right]$$

we see that (25) will not carry any transverse infinities if $\rho(W) \sim W^\epsilon$ with $\epsilon < -1$. This is visibly true in the Landau gauge where (22) is the zeroth solution, but in fact it is also true in other gauges as well because there is asymptotic stability. It may be of interest to spell out the first-order photon propagator to two extreme limits.

(i) As $k^2 \to 0$,

$$\int \mathrm{d}W \rho(W)(\Pi(k^2, W^2) - \Pi(0, W^2)) \to \frac{e^2}{60\pi^2}\int \frac{k^2 \rho(W)\,\mathrm{d}W}{W^2} = \frac{e^2 k^2}{60\pi^2 m^2}\left[1 + \left(\frac{m^2}{\mu^2}\right)^{2\xi} \Gamma(2-\xi)\Gamma(2-\xi)\Gamma(1+2\xi)\right].$$

Therefore

$$D^{(1)-1}_{\text{transv}} \to k^2\left[1 + \frac{e^2 k^2}{60\pi^2 m^2}\left(\frac{1 + (m^2/\mu^2)^{2\xi}\Gamma(2-\xi)\Gamma(2-\xi)\Gamma(1+2\xi)}{1 + (m^2/\mu^2)^{2\xi}\Gamma(1-\xi)\Gamma(1-\xi)\Gamma(1+2\xi)}\right)\right].$$

(ii) As $k^2 \to \infty$

$$\int \mathrm{d}W \rho(W)\Pi(k^2, W^2) \to +\frac{e^2}{12\pi^2}\int \rho(W) \ln\left(-\frac{k^2}{W^2}\right)\mathrm{d}W.$$

Because $\int \mathrm{d}W \rho(W) \ln W$ is finite like Z_ψ, in the Landau gauge, there remains a logarithmic dependence on k^2 in $D^{(1)}$; this shows that Z_A^{-1} is logarithmically infinite in this next order. All in all, the first-order corrections are manageable and cannot greatly affect the propagator $S^{(2)}$ when we go to the next order of iteration.

More significant probably are the transverse parts to the vertex part $\Gamma^{(1)}$ that enter into (9) at the next level. It is conceptually simpler to deal with this equation in the form (5′) whereupon

$$\Gamma^{(1)}_{\mu}(p, p-k)Z_{\psi}^{-1} = \gamma_{\mu} - ie^2 \int \bar{d}^4p' \, \gamma_\lambda S^{(0)}(p')\Gamma^{(0)}_{\nu\mu}(p'k'; pk)D^{(0)\lambda\nu}(k). \tag{26}$$

Since the longitudinal part $k \, . \, \Gamma^{(1)}$ equals $k \, . \, \Gamma^{(0)}$, it is already known; and so is, of course, the $k \to 0$ limit via the differential Ward identity. We have not made a detailed study of (26) since the number of kinematic terms that can turn up lead to very complicated expressions. However we may note that if one goes to the electron mass shell, then because $\Gamma^{(0)}$ reduces to the Born term, the calculation is exactly the order α correction of the form factor in QED and cannot fail to reproduce $g-2$ in this order. However we have no reason to believe that $\Gamma^{(2)}$ includes the α^2 correction of the form factor because $\Gamma^{(1)}_{\mu\nu}$ may have no direct connection with Born graphs.

To summarize our work thus far: we have found a *gauge covariant* solution of the Dyson–Schwinger equation for the electron propagator and it provides the basis of a subsequent iteration scheme† to yield all the remaining Green functions. The solution has the merit of exactly satisfying gauge identity (1) and in that sense is a significant generalization of the Baker *et al* (1967) solution as well as the parallel later work that has consisted in replacing Γ by the bare vertex γ in the equation, including the work on self-consistent dynamical symmetry breaking.

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† Though whether the iterations converge to well defined answers is a matter of speculation.